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# Parameter selection for aeroengine transient state gas path analysis

# **Original article**

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# Keywords:

parameter selection; aeroengine; transient state; gas path analysis; smearing effect

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# Abstract

Compared with steady state data, engine transient state data cover a wider working scope of components, more complex operating conditions, and therefore can provide more abundant information regarding engine health status. To make the fullest use of engine transient data, this paper presents the methods on how to choose the proper measurements and health parameters when constructing an aeroengine gas path analysis system.

The measurement selection based on sensitivity analysis is conducted, compared to which an effortless measurement selection method termed Minimum Identifiability Loss (MIL) is proposed. Aiming to reduce the estimation uncertainty, the analysis of maximal linearly independent group is presented to select optimal health parameters under the condition of limited measurements. Finally, the component health status identifiability analysis base on transient data also gives a good explanation to gas path analysis "smearing effect".

# Introduction

Performance degradation of aeroengines is an inevitable problem in their regular operations. A timely and accurate grasp of the health status of each component is the basis of formulating a reasonable maintenance plan to ensure flight safety and maintain the economic affordability of engine operation. So far, nonlinear model-based steady state gas path analysis is still a necessary means for aeroengine health monitoring. Compared with steady state data, engine transient state data cover a wider working scope of components, more complex operating conditions, and can provide more abundant engine health information. Therefore, the research of transient state gas path analysis has important theoretical significance and engineering application value.

Parameter selection is an unavoidable fundamental problem in gas path analysis. Therefore, almost synchronous with the introduction of aeroengine gas path analysis, Urban (1975) proposed to match the measured quantities and health parameters based on the inverse of the Influence Coefficient Matrix (ICM), that is, the fault coefficient matrix. Subsequent research focused on the discussion of ICM. Stamatis et al. (1992) proposed a method to quickly select health parameters based on the projection of the health parameter vector on the corresponding singular vector of the Jacobian matrix, and determined the measurement subset according to the results of sensitivity analysis. Provost (1994) proposed a method to distinguish redundant measurements based on parameter correlation analysis. España (1993) pointed out that the necessary condition for an observable system is that the number of measurements is no less than that of the health parameters to be determined. Grönstedt (2002) judged the optimal combinations of health parameters in different dimensions by analysing the condition number of the system Hessian matrix. Ogaji et al. (2002) utilized measurement sensitivity to determine their ability of quantifying implanted faults, and uses the Root Mean Square Error (RMSE) of the estimated results for measurement set evaluation. Mathioudakis and Kamboukos (2006) proposed a method to determine measurements, health parameters and the number of operating points by analysing the condition numbers of the Jacobian matrices based on the data of single or multiple operating points. McCusker and Danai (2010) obtained parameter signatures by performing wavelet analyses on the small disturbance output signals during transient process, and selected the necessary measurements accordingly. In the framework of Systematic Sensor Selection Strategy (S4), Sowers et al. (2008) proposed to determine the optimal combination of measuring points according to their merit value, and used the deviation of the measurements or their root mean square to judge the detectability of each health parameter. Borguet and Léonard (2008) used the figure of merit, which is defined based on the characteristic quantity of the system Fisher information matrix, to select the optimal measurement set, and employed the sensitivity index to evaluate the set. Simon and Rinehart (2016) commended a measurement selection method on the basis of statistical theory.

Although the above-mentioned research on the selection of gas path analysis parameters has improved the accuracy of engine condition monitoring, they are all based on engine steady state and cannot provide sufficient theoretical support for engine transient state gas path analysis. In this paper, we present the methods for system construction of transient state gas path analysis and their applications.

# Methodology

#### Nonlinear gas path analysis

Aeroengine nonlinear condition monitoring model can be expressed as the following nonlinear equations.

$$\dot{x}(t) = f(x(t), u(t), \theta(t)) \tag{1}$$

$$z(t) = g(x(t), u(t), \theta(t))$$
(2)

The components of the health parameter vector  $\boldsymbol{\theta}$  in the above equations are generally defined by the ratio of the performance parameters of the degraded or malfunctioning engine components to those of the clean engine components, and in the form of

$$\theta_{i,\mathrm{d}} = \frac{X_{i,\mathrm{d}}}{X_{i,\mathrm{c}}} \quad i = 1, \ \dots, p \tag{3}$$

In the numerical calculation of health parameter estimation, Equation 2 can be linearized at the currently estimated value of the health parameters  $\hat{\theta}$ .

$$z_{\rm d}(t) \approx \hat{z}(t) + J(t, \hat{\theta}(t)) \left(\theta_{\rm d}(t) - \hat{\theta}(t)\right) \tag{4}$$

$$\hat{z}(t) = g(x(t), u(t), \theta(t)) \tag{5}$$

where the Jacobian matrix

$$J(t, \hat{\theta}(t)) = \begin{bmatrix} \frac{\partial z_1(t, \theta(t))}{\partial \theta_1} & \cdots & \frac{\partial z_1(t, \theta(t))}{\partial \theta_p} \\ \vdots & \ddots & \vdots \\ \frac{\partial z_q(t, \hat{\theta}(t))}{\partial \theta_1} & \cdots & \frac{\partial z_q(t, \hat{\theta}(t))}{\partial \theta_p} \end{bmatrix}$$
(6)

Equation 4 can be written in residual form as

$$\Delta \hat{z}(t) = J(t, \,\hat{\theta}(t)) \,\,\Delta \hat{\theta}(t) \tag{7}$$

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where the estimated measurement residual (in relative form)

$$\Delta \hat{z}_{j}(t) = \frac{z_{j,d}(t) - \hat{z}_{j}(t)}{z_{j,d}(t)} \quad j = 1, \dots, q$$
(8)

and the estimated health parameter residual

$$\Delta\hat{\theta}_i(t) = \theta_{i,d}(t) - \hat{\theta}_i(t) \quad i = 1, \dots, p$$
(9)

Then in the *k*th iterative calculation, we have

$$\hat{\theta}^{k+1}(t) = \hat{\theta}^{k}(t) + J^{-1}(t, \,\hat{\theta}^{k}(t))\Delta\hat{z}^{k}(t)$$
(10)

#### Parameter selection

#### Sensitivity analysis

When a certain health parameter degrades, measurements at different cross-sections or with different properties deviate with different magnitudes or even different signs. Therefore, the traditional method (Urban, 1975) believes that those deviations in the measurements will indicate their ability of identifying this health parameter.

Supposing  $\theta_c$  is the health parameter of a healthy engine, then  $\theta_c = \mathbf{I}$  can be generated from Equation 3. Let *h* be a small percentage constant (say 1%) and define the incremental vector of health parameters as

$$\delta\theta_i = \left(\begin{array}{cccc} \overset{i-1}{\underbrace{0, \dots, 0}}, & h, & \overbrace{0, \dots, 0}^{n-i} \end{array}\right) \quad i = 1, \dots, p \tag{11}$$

Substituting  $\theta_c$  and  $\delta \theta_i$  into Equation 2, we obtain

$$z_{\rm c}(t) = g(x(t), u(t), \theta_{\rm c}(t)) \tag{12}$$

$$z_i(t) = g(x(t), u(t), \theta_c(t) + \delta \theta_i) \quad i = 1, \dots, p$$
(13)

Combining Equations 12 and 13, then we obtain the sensitivity of measurement j to health parameter i at sampling time t as

$$\delta z_{j,i}(t) = \frac{z_{j,i}(t) - z_{j,c}(t)}{z_{j,c}(t)} \quad j = 1, \dots, q; \quad i = 1, \dots, p$$
(14)

Equation 14 can also be written in matrix form as

$$S(t) = \begin{bmatrix} \delta z_{1,1}(t) & \cdots & \delta z_{1,p}(t) \\ \vdots & \ddots & \vdots \\ \delta z_{q,1}(t) & \cdots & \delta z_{q,p}(t) \end{bmatrix}$$
(15)

# Identifiability analysis

When Equation 15 is full column rank, the health parameter corresponding to each column is identifiable. In this paper, the singular value decomposition method is used to obtain the rank of the matrix. Then the matrix in Equation 15 can be decomposed as

$$S(t) = U(t)\Sigma(t)V^{\mathrm{T}}(t)$$
(16)

The diagonal elements of  $\Sigma(t)$  are the singular values of S(t) in descending order, while the number of non-zero singular values is the rank. Each column of the unitary matrix V(t) represents one identifiable direction in the health parameter space under the current measurement settings, and the corresponding singular value indicates its degree of identifiability. The larger the singular value is, the greater the degree of stretching on the corresponding direction will be when transforming from the health parameter space to the measurement space, and then the easier to identify small changes of the health parameters. The above-mentioned health parameter space can be represented by a hyper-ellipsoid in a dimension equal to the number of health parameters. The singular vector and the reciprocal of the singular value correspond to the direction and relative length of the hyperellipsoid axis, respectively. A two-dimensional health parameter space is schematically illustrated in Figure 1.

It is obvious that the longest axis of the hyper-ellipsoid determines the maximum span of the health parameter space, which corresponds to the upper bound of system uncertainty. Therefore, the smallest singular value  $\sigma_{\min}(t)$  can be used to measure system identifiability.

# Indices definition

To describe the sensitivity of the measurements from the perspective of a whole process, it is defined here that the Root Mean Square (RMS) sensitivity of measurement j to health parameter i as

$$\delta \bar{z}_{j,i} = \sqrt{\frac{\sum_{k=1}^{N} \delta z_{j,i}^2(T_k)}{N}} \quad j = 1, \dots, q; \quad i = 1, \dots, p$$
(17)

Two more indices are defined to describe the analysing ability of the engine health status of a given measurement set. One is the Process Averaging Identifiability (PAI) in Equation 18 to indicate its identifiability, the other is Process Averaging Root Mean Square Error (PARMSE) in Equation 19 to indicate its accuracy.

$$\bar{\sigma}_{\min} = \frac{\sum_{k=1}^{N} \sigma_{\min}(T_k)}{N} \tag{18}$$

$$\bar{e} = \frac{\sum_{k=1}^{N} \sqrt{\left(\sum_{i=1}^{p} \left(\theta_{i,\text{implanted}} - \hat{\theta}_{i}(T_{k})\right)^{2}\right)/p}}{N}$$
(19)



Figure 1. Two-dimensional health parameter identification space.

# Results and discussion

In this section, two problems will be discussed separately: measurement selection when there is instrumental redundancy, and health parameter selection when the number of measurements is limited.

# The engine model

The engine model used for parameter selection is a separated flow turbofan engine. Figure 2 depicts the stations where outputs are simulated. A slam acceleration process using the fuel schedule shown in Figure 3 is chosen for the analysis.

# Measurement selection

Supposing there is instrumental redundancy. The candidate measurements, the health parameters to be estimated and their implanted deviations are listed in Table 1 (subscripts are described in Figure 1), where  $\Gamma$  and  $\eta$  represent the corrected flow rate and efficiency of the corresponding component. Sensor dynamics and noise are ignored for simplicity in this study.

To estimate all health parameters listed in Table 1, simultaneously, and avoid unnecessary instrument installation, we need to select 10 measurements among the 12 candidates in a way that is most conducive to engine health status estimation. There will be two ways involved in this paper for measurement selection: one is sensitivity analysis-based, and the other is identifiability analysis-based.

# Sensitivity analysis-based measurement selection

Equation 17 is used to calculate the RMS sensitivity of each measurement corresponding to each health parameter, and the results are listed in Table 2. Data from 80 sampling points (N = 80) in this transient process are chosen for the analysis, and the sampling interval is 0.05 s. According to the data of each column in Table 2, the descending sequence of measurement sensitivity to each health parameter can be obtained (Table 3).

Because the 6 measurements ( $P_{25}$ ,  $N_{LP}$ ,  $P_3$ ,  $N_{HP}$ ,  $P_{45}$  and  $T_5$ ) in the first column of Table 3 have the highest RMS sensitivity to the corresponding health parameter, therefore they are firstly selected. Then we need to determine the optimal measurement set among the remaining  $C_6^4 = 15$  sets. The PAI and PARMSE obtained for each set according to Equations 18 and 19 are listed in Table 4.

Since every parameter estimation process takes 90% of the implanted deviation in Table 1 as the initial value, the initial PARMSE of all estimations is  $1 \times 10^{-3}$ . Results in Table 4 show that set 1–4, 7 and 8 are the optimal measurement sets, which improve the PARMSE by about two orders of magnitude.

# Identifiability analysis-based measurement selection

Inevitably, measurement selection results based on sensitivity analysis is susceptible to subjective factors. Therefore, a large number of validation calculations are required. On the other hand, traditional identifiability analysis-based methods usually adopt exhaustive strategy, so the amount of calculation would also be



Figure 2. Schematic diagram of the simulated separated flow turbofan engine.



#### Figure 3. Fuel schedule.

considerable. This paper proposes the Minimum Identifiability Loss (MIL) method, which uses identifiability as the objective criterion and adopts exclusive strategy. Thus it avoids the influence of subjective factors and reduces the amount of calculations.

The procedures of MIL are:

- 1. Calculate the PAI of the measurement set which includes all 12 candidate measurements in Table 1.
- 2. Calculate the PAI of the measurement set consist of 11 candidate measurements in Table 1, which means a certain measurement will be excluded each time.

No.	Candidate Measurement	Health Parameter	Implanted Deviation $\Delta  heta_{i, d}$
1	P <sub>13</sub>	$\Gamma_{Fan}$	-1%
2	T <sub>13</sub>	$\eta_{Fan}$	-1%
3	P <sub>25</sub>	$\Gamma_{ m LPC}$	-1%
4	T <sub>25</sub>	$\eta_{LPC}$	-1%
5	N <sub>LP</sub>	$\Gamma_{HPC}$	-1%
6	P <sub>3</sub>	$\eta_{ m HPC}$	-1%
7	T <sub>3</sub>	$\Gamma_{ m HPT}$	1%
8	N <sub>HP</sub>	$\eta_{ m HPT}$	-1%
9	P <sub>45</sub>	$\Gamma_{LPT}$	1%
10	T <sub>45</sub>	$\eta_{ m LPT}$	-1%
11	P <sub>5</sub>		
12	<i>T</i> <sub>5</sub>		

Table 1. Candidate measurements and health parameters considered.

	$arGamma_{Fan}$	$\pmb{\eta}_{Fan}$	$arGamma_{ t LPC}$	$\eta_{ ext{LPC}}$	$\Gamma_{HPC}$	$\eta_{ ext{HPC}}$	$arGamma_{HPT}$	$\eta_{ m HPT}$	$arGamma_{LPT}$	$\eta_{LPT}$
P <sub>13</sub>	3.0984	2.4976	0.1972	0.6576	0.3708	1.2753	0.2788	1.6462	0.8084	3.3163
T <sub>13</sub>	0.2558	0.1255	0.0827	0.2213	0.1006	0.4394	0.0885	0.5649	0.2828	1.2326
P <sub>25</sub>	2.1836	2.3877	1.9680	0.4314	0.3496	0.9078	0.2382	1.2233	1.4424	3.2065
T <sub>25</sub>	0.5708	0.6553	0.1248	1.5240	0.0856	0.3404	0.0659	0.4436	0.2023	0.8911
N <sub>LP</sub>	3.5694	3.4729	0.2580	1.1043	0.7716	1.9664	0.4874	2.5393	1.3239	4.7548
P <sub>3</sub>	0.7290	0.5953	0.7808	0.7742	1.7717	5.6814	9.4521	9.1552	2.7518	0.7916
T <sub>3</sub>	0.5034	0.5208	0.4544	1.3850	1.5252	3.5963	2.7498	3.8819	1.4579	0.6636
N <sub>HP</sub>	1.0540	1.1456	0.5395	0.8122	3.6004	4.2324	0.9969	5.8179	2.3801	1.4922
P <sub>45</sub>	2.9968	3.0419	1.3736	2.2731	2.3946	9.7674	1.7895	12.9151	6.8005	4.0136
T <sub>45</sub>	0.8444	0.7826	0.5869	0.9755	1.2502	5.5113	0.7792	7.5278	2.5939	1.0430
P <sub>5</sub>	0.5711	0.5544	0.5155	0.1592	0.6523	2.4931	0.4757	3.2758	1.8763	2.5666
$T_5$	0.3406	0.3728	0.8777	1.6603	1.6960	7.8476	1.1680	10.4820	1.2244	5.5336

Table 2. RMS sensitivity of the measurements  $(10^{-3})$ .

Table 3. Descending sequence of measurement sensitivity.

Health parameter	Measu	rement	ment							
	1	2	3	4	5					
$\Gamma_{\rm Fan}$	N <sub>LP</sub>	P <sub>13</sub>	P <sub>45</sub>	P <sub>25</sub>	N <sub>HP</sub>					
$\eta_{Fan}$	$N_{\rm LP}$	P <sub>45</sub>	P <sub>13</sub>	P <sub>25</sub>	N <sub>HP</sub>					
$\Gamma_{\rm LPC}$	P <sub>25</sub>	P <sub>45</sub>	$T_5$	P <sub>3</sub>	T <sub>45</sub>					
$\eta_{ ext{LPC}}$	P <sub>45</sub>	$T_5$	T <sub>25</sub>	T <sub>3</sub>	T <sub>45</sub>					
$\Gamma_{\rm HPC}$	N <sub>HP</sub>	P <sub>45</sub>	P <sub>3</sub>	$T_5$	T <sub>3</sub>					
$\eta_{ ext{HPC}}$	P <sub>45</sub>	$T_5$	P <sub>3</sub>	T <sub>45</sub>	N <sub>HP</sub>					
$\Gamma_{\rm HPT}$	P <sub>3</sub>	$T_3$	P <sub>45</sub>	$T_5$	N <sub>HP</sub>					
$\eta_{ m HPT}$	P <sub>45</sub>	$T_5$	P <sub>3</sub>	T <sub>45</sub>	N <sub>HP</sub>					
$\Gamma_{\rm LPT}$	P <sub>45</sub>	P <sub>3</sub>	T <sub>45</sub>	N <sub>HP</sub>	$P_5$					
$\eta_{ m LPT}$	$T_5$	N <sub>LP</sub>	P <sub>45</sub>	P <sub>13</sub>	P <sub>25</sub>					

Set	P <sub>13</sub>	T <sub>13</sub>	P <sub>25</sub>	T <sub>25</sub>	N <sub>LP</sub>	<i>P</i> <sub>3</sub>	T <sub>3</sub>	N <sub>HP</sub>	P <sub>45</sub>	T <sub>45</sub>	P <sub>5</sub>	$T_5$	$\bar{\sigma}_{ m min}$ (10 <sup>-4</sup> )	ē
1	$\times$		$\times$	$\times$	×	$\times$	$\times$	$\times$	$\times$		$\times$	$\times$	6.38	$1.30 \times 10^{-5}$
2	$\times$		×	$\times$	×	$\times$	$\times$	$\times$	$\times$	$\times$		$\times$	6.32	$1.31 \times 10^{-5}$
3	$\times$	$\times$	$\times$	$\times$	×	$\times$	$\times$	$\times$	$\times$			$\times$	3.74	$1.32 \times 10^{-5}$
4	$\times$		$\times$		$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	2.12	$1.31 \times 10^{-5}$
5		$\times$	$\times$	$\times$	×	$\times$	$\times$	$\times$	$\times$		$\times$	$\times$	1.01	$8.98 \times 10^{-4}$
6		$\times$	$\times$	$\times$	×	$\times$	$\times$	$\times$	$\times$	$\times$		$\times$	0.90	$6.52 \times 10^{-4}$
7	$\times$	$\times$	×		×	$\times$	$\times$	$\times$	×	×		$\times$	0.49	$1.32 \times 10^{-5}$
8	$\times$		×	$\times$	×	$\times$		$\times$	×	$\times$	$\times$	$\times$	0.15	$1.37 \times 10^{-5}$
9		$\times$	×		×	$\times$	$\times$	$\times$	×	$\times$	$\times$	$\times$	0.12	$4.22 \times 10^{-5}$
10			×	$\times$	×	$\times$	$\times$	$\times$	×	$\times$	$\times$	$\times$	0.06	$4.32 \times 10^{-1}$
11		×	×	$\times$	×	$\times$		$\times$	×	$\times$	$\times$	$\times$	0.04	$3.85 \times 10^{-4}$
12	$\times$	$\times$	×		×	$\times$	$\times$	$\times$	×		$\times$	$\times$	0.04	$4.72 \times 10^{-5}$
13	$\times$	×	×		×	$\times$		$\times$	×	$\times$	$\times$	$\times$	0.00	$2.40 \times 10^{-2}$
14	$\times$	$\times$	$\times$	$\times$	×	$\times$		×	$\times$		$\times$	$\times$	0.00	$6.71 \times 10^{-5}$
15	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$		$\times$	$\times$	$\times$		$\times$	0.00	5.21 × 10 <sup>-5</sup>

Table 4. Results of sensitivity analysis-based method.

3. Find the *q-p* measurement sets with the highest PAI in step 2 and exclude the measurements that are excluded by these sets, then the remaining candidate measurements will constitute the optimal measurement set.

The previous measurement selection task is then handled again by the MIL method, the results are shown in Table 5.

In Table 5, set 1 contains all 12 candidate measurements, while set 2 to 13 contain 11 candidate measurements. Results show that set 1 has the highest PAI, and the PAI of the other sets decline diversely, among which set 2 and set 3 have the least decline relative to set 1. Therefore, compared to other candidate measurements,  $T_{13}$  and  $T_{45}$  have the least contribution to system identifiability and are excluded firstly. Thereby we obtain an optimal measurement set as same as the set 1 in Table 4.

Compared with sensitivity analysis-based method and traditional identifiability-based method adopting exhaustive strategy, the MIL method adopts a simpler selecting logic, and avoids the calculation of a large number of measurement combinations. Especially, when the number of the candidate measurements increases, the calculation amount of the MIL method would only grow linearly.

# Health parameter selection

Supposing the number of measurements is limited. A typical 7 gas path measurements set is listed in Table 6.

It is not possible to estimate all the health parameters shown in Table 1 simultaneously using the measurement set given in Table 6, obviously. Therefore, we have to select the health parameter combinations that can be better identified. The health parameter selection problem considering here has the following features:

1. Due to the varying engine nonlinearity with the operating process, the nature and quantity of the health parameters that can be estimated at different sampling points may also change.

Set	P <sub>13</sub>	T <sub>13</sub>	P <sub>25</sub>	T <sub>25</sub>	$N_{\rm LP}$	P <sub>3</sub>	T <sub>3</sub>	N <sub>HP</sub>	P <sub>45</sub>	T <sub>45</sub>	P <sub>5</sub>	$T_5$	$\bar{\sigma}_{ m min}$ (10 <sup>-4</sup> )
1	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	×	$\times$	$\times$	$\times$	×	×	6.55
2	$\times$		$\times$	$\times$	$\times$	×	×	$\times$	$\times$	$\times$	×	×	6.49
3	$\times$	$\times$	$\times$	$\times$	$\times$	×	×	$\times$	$\times$		×	×	6.45
4	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	×	$\times$	$\times$	$\times$		×	6.35
5	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	×	$\times$	$\times$	$\times$	×		5.83
6	$\times$	$\times$	$\times$		$\times$	$\times$	×	$\times$	$\times$	$\times$	×	×	2.18
7	$\times$	$\times$	$\times$	$\times$	$\times$	×	×		$\times$	$\times$	×	×	2.11
8		$\times$	$\times$	$\times$	$\times$	×	×	$\times$	$\times$	$\times$	×	×	1.02
9	$\times$	$\times$	$\times$	$\times$	$\times$		×	$\times$	$\times$	$\times$	×	×	0.57
10	$\times$	$\times$		$\times$	$\times$	×	×	$\times$	$\times$	$\times$	×	×	0.53
11	$\times$	$\times$	$\times$	$\times$		×	×	$\times$	$\times$	$\times$	×	×	0.24
12	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$		$\times$	$\times$	$\times$	0.14
13	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$		$\times$	×	$\times$	$\times$	$\times$	0.07

Table 5. Results of MIL method.

#### Table 6. Typical gas path measurements

No.	1	2	3	4	5	6	7
Measurement	P <sub>13</sub>	T <sub>13</sub>	$N_{\rm LP}$	P <sub>3</sub>	T <sub>3</sub>	N <sub>HP</sub>	$T_5$

2. Because the selection process involves changes in the health parameter space, the applicability of traditional sensitivity analysis-based and identifiability analysis-based methods would be greatly restricted.

According to the aforementioned identifiability theory, what we need to do is to construct system matrices with full column rank in the form of Equation 15. Thereby, this paper proposes a health parameter selection method based on maximal linearly independent group analysis, and its procedure is as follows:

- 1. Construct the  $7 \times 10$  system matrix at each time point based on Equation 14, and apply Equation 16 to obtain their rank r(t).
- 2. Construct the  $7 \times r(t)$  subsystem matrices by different health parameter combinations, and apply Equation 16 to obtain their ranks and smallest singular values.
- 3. Then the health parameter combinations corresponding to the full rank subsystem are regarded as feasible combinations.

Taking the results at sampling point t = 1 s (Table 7) as an example.

Results show that the maximum number of identifiable health parameters at t = 1 s is 7, and there are 10 feasible combinations, among which combination 3 has the highest identifiability ( $\sigma_{min}$ ) and a comparatively lower estimation error, but combination 1 has the least identifiability and estimation accuracy. Thereby we can put the health parameter combination with the highest identifiability at each sampling point into one best identifiability

Combination	$arGamma_{Fan}$	$\eta_{Fan}$	$\Gamma_{ t LPC}$	$\eta_{ ext{LPC}}$	$\Gamma_{HPC}$	$\eta_{ ext{HPC}}$	$arGamma_{HPT}$	$\eta_{ ext{HPT}}$	$\Gamma_{\rm LPT}$	$\eta_{ ext{LPT}}$	$\sigma_{\rm min}$ (10 <sup>-4</sup> )	RMSE
1	$\times$	$\times$		$\times$	$\times$	×	$\times$			$\times$	1.6202	$1.68 \times 10^{-7}$
2	$\times$	$\times$		$\times$	$\times$		$\times$	×	×		2.4251	$4.82 \times 10^{-9}$
3	$\times$	$\times$		$\times$	$\times$		$\times$	×		$\times$	3.4983	$3.56 \times 10^{-9}$
4	$\times$	$\times$		$\times$	$\times$		$\times$		×	$\times$	3.2975	$2.95 \times 10^{-9}$
5	$\times$	$\times$		$\times$		×	$\times$	×	$\times$		2.0767	$5.62 \times 10^{-9}$
6	$\times$	$\times$		$\times$		×	$\times$	×		$\times$	2.6096	$5.53 \times 10^{-9}$
7	$\times$	$\times$		$\times$		×	$\times$		$\times$	$\times$	2.4780	$4.73 \times 10^{-9}$
8	×	$\times$			$\times$	×	$\times$	×	×		2.5089	$8.15 \times 10^{-9}$
9	$\times$	$\times$			$\times$	×	$\times$	×		$\times$	3.4910	$7.90 \times 10^{-9}$
10	$\times$	$\times$			$\times$	$\times$	$\times$		$\times$	$\times$	3.3896	$2.52 \times 10^{-9}$

Table 7. Full rank subsystem results at sampling point t = 1 s

matrix as shown in Figure 4. The order of the health parameters in Figure 4 are consistent with Table 1. Colour blue and white represent "include" and "not include", respectively.

It also shows that there are slightly differences of the maximum number of identifiable health parameters at different sampling points, that is, 7 for the majority of the sampling points except 6 at t = 1.7 s and t = 1.8 s.

# Smearing effect analysis

Considering the identifiability of the engine health status on a component level by the measurement set shown in Table 6. According to the health parameters listed in Table 1, the health status of each main gas path component is described by two health parameters. So the maximum number of components that can be analysed simultaneously by the measurement set shown in Table 6 is 3 at each sampling point. Therefore, theoretically, the number of identifiable component combinations is  $C_5^1 + C_5^2 + C_5^3 = 25$ .

By constructing the subsystem matrix of the 25 component combinations at a certain sampling point and calculating their rank, we can obtain the identifiability of these subsystems. The results are shown in Figure 5. The left part of Figure 5 shows the component combinations, with colour black and white represent "include" and "not include", respectively. The right part of Figure 5 shows the identifiability of the corresponding component combination, with colour blue and white represent "identifiable" and "unidentifiable", respectively.



Figure 4. Health parameter best identifiability matrix.



Figure 5. Component identifiability matrix.

Figure 5 shows that the identifiability of the component combinations vary with sampling time as well as the number of components. In general, an increase in the number of components leads to a decrease in identifiability.

Take combination 10 and combination 15 as examples, the estimation results of these two combinations are shown in Figure 6. The number of combinations and sampling points are consistent with Figure 5, and the colour green, yellow, and red represent "converged with high precision", "converged with low precision" and "diverged", respectively.

Combining the results in Figures 5 and 6, unsatisfactory estimation result only occurs when the corresponding component combination is unidentifiable. Take the results of combination 10 at t = 0.9 s and combination 15 at t = 0.1 s as examples (Figure 7).

It is shown that there are large deviations in both cases, which means "smearing effect" occurs. Aretakis et al. (2002) and Mathioudakis et al. (2004) firstly pointed out this phenomenon in fault diagnosis, and they also provided a solution for this problem, that is, addition of measured quantities between the corresponding components. Based on the above results, it can be concluded that "smearing effect" is caused by the measurement set as well as the engine nonlinearity at a certain operating point, which means only when the combination of these two factors leads to a singular system, may this phenomenon occur. From the perspective of system identification, "smearing effect" means an increase in system uncertainty.



Figure 6. Estimation results of combination 10 and 15.



Figure 7. Estimation results of combination 10 at t = 0.9 s (left) and combination 15 at t = 0.1 s (right).

# Conclusions

The methods on how to choose the proper measurements and health parameters when constructing an aeroengine gas path analysis system using transient data have been presented. The traditional sensitivity analysis-based measurement selection has been conducted, and a new measurement selection method adopting exclusive strategy has been proposed. A health parameter selection method based on maximal linearly independent group analysis has been proposed. The root cause of "smearing effect" has been revealed. The applicability of the proposed method has been demonstrated on a separated flow turbofan engine, and the results show that:

- 1. The indices defined in this paper is effective.
- 2. The MIL method adopts a simple selecting logic, and avoids the calculation of a large number of measurement combinations.
- 3. Maximal linearly independent group analysis is effective to determine the optimal combination of health parameters at each sampling point under the condition of limited measurements.
- 4. "Smearing effect" is caused by measurement settings as well as engine nonlinearity, and it occurs only when the combination of these two factors leads to a singular system at a specific working point.

# Nomenclature

 $\overline{e}$ 

r Tt u x z Γ η θ

h	small percentage constant
MIL	minimum identifiability loss
Ν	rotational speed
P	total pressure
р	number of health parameters
9	number of measurements
RMSE	root mean square error
r	rank
Τ	total temperature
t	time
и	engine input vector
x	engine performance vector
z	measurement vector
Γ	health parameter (corrected flow rate)
η	health parameter (efficiency)
θ	health parameter vector

process averaging root mean square error

process averaging identifiability  $\bar{\sigma}_{
m min}$ 

# Subscript

- С clean engine
- degraded engine d
- HP high pressure
- low pressure LP

# Competing interests

Kun Yang declares that he has no conflict of interest. Qiuye Tu declares that he has no conflict of interest. Yunhao Zeng declares that he has no conflict of interest. Jing Xuan declares that she has no conflict of interest.

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